# "Novel formulation of the quadrupole equation for potential stellar gravitational-wave power estimation"

**by Robert M. L. Baker, Jr.**

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## Peer-review Referee's Comments:

The paper is excellent and of necessary relevance to the field of gravitational wave astronomy. The author derives and discusses an important, novel way to calculate the estimated gravitational wave power from an astronomical system using a new novel interpretation for the moment of inertia term inside the kernel of Einstein's power equation. This new method reduces the complexity and greatly simplifies calculating the theoretical gravitational wave power estimate. The author successfully demonstrated that his new method reproduces the published (predicted and experimental) result for the Hulse-Taylor (1994) observation of PSR 1913+16. The author also applies his new method to other astronomical systems of interest for gravitational wave studies.

There are typographical errors that I noted in the manuscript, and these should be corrected. These will be noted in the below comments.

I did note possible formatting errors within the paper .These were the proper formatting of the list of references and of the author's citation of references within the text. I found the latter to be inconsistent within the paper. For example, sometimes the author cites "A. Einstein (1918)."Other times he cites "(Einstein 1918)," but without the "A." and without a comma after "Einstein." And other times he cites "(Gallo et al. 2005)" again without first name initials or the comma. I am not familiar with your journal's particular reference citation format, but hope that you will address these with the author.

In the sentence that follows equation (2), the author states that kappa is defined to be G (universal gravitational constant), yet there is a factor of G already present in the numerator of the fraction inside the parenthesis. This is confusing because there should only be one factor of G in equation (1). Is this a typographical error? In the 2nd paragraph, 4th sentence of the Discussion section there are embedded parentheses that cause grammatical confusion. This sentence could be more correctly written as: [The midpoint or GW focus at the two star's center of mass can be unrelated to any stellar-system's (that is, a large number of stellar masses such as a galaxy's) center of mass.] Would you agree?

There are other instances of embedded parentheses where the author could eliminate confusion if he used the embedded bracket rule: {[ ( ) ] }. But the author probably needs to check with the editor on whether the journal has particular requirements in this regard.

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## Peer-review Referee's Comments:

The paper is a relevant and important contribution to gravitational-wave astrophysics. Although the quadrupole formalism expressed by Eqs. (9) and (16) involves no new relativistic Physics, it is a fresh a and innovative formulation and his derivation checks out as correct. I would, however, recommend that his novel quadrupole formulation be mentioned as having application to laboratory GW generation. It clearly exhibits the importance of a large impulsive force change over a very short time interval acting at radiators far apart to generate significant GWs and might be accomplished in a laboratory setting. I also observe that unlike orbiting or in-spiraling black holes the GW polarization does not rotate, but is fixed. This effect could be utilized to advantage in detection. It is also to be noted that the allowance for large r compared to GW wavelength is <u>not</u> prohibited from any theory that I have found and the quadrupole formalism approximately holds. Thus it could be emphasized in the paper that the quadrupole formalism applies to intergalactic distances between GW radiators, thereby releasing enormous GWs at a focus midway between the radiators. I recommend publication.

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# Novel Formulation of the Quadrupole Equation for Potential Stellar Gravitational-Wave Power Estimation

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**Abstract.** A novel formulation of the quadrupole equation for potential stellar gravitational-wave power estimation is derived. The derivation commences with the classical Einstein quadrupole formalism and then utilizes Newton's second law to establish a simplified formulation involving the radius of gyration of a mass or system of masses involving a pair of massive stars either on orbit about one another, or otherwise separated, or a star with a dumbbell-like or aspherical mass distribution and an impulsive force acting on the mass or masses in order to estimate the power of a gravitational wave that is generated. A numerical example, based upon the well-known gravitational-wave power observed to be generated by PSR 1913+16, is utilized to test the formulation. Potential applications to stellar jets, including stellar-black-hole produced jets, are cited as examples of the potential applications of the novel quadrupole formulation. It is suggested that the gravitational waves, generated by the applications suggested, might be detected by the proposed space-based Laser Interferometer Space Antenna or LISA.

**Key words:** Gravitational waves, ISM jets and outflows



## 1. Introduction

Recent analyses of stellar-black-hole dark jets (Gallo et al. 2005) as well as the models of the ubiquitous stellar jets bring to mind the possibility that there exist gravitational wave (GW) generation by these forceful events. If one could determine the GW-wave power generated in a simple fashion, then estimates could be made of their detectability by the proposed space-based Laser Interferometer Space Antenna or LISA. The derivation of the new formulation commences with the classical Einstein quadrupole formalism and then utilizes Newton's second law to establish a simplified formulation involving the radius of gyration of a mass or system of masses involving a pair of massive stars either on orbit about one another, or otherwise separated, or a star with a dumbbell-like mass distribution and an impulsive force acting on the mass or masses in order to estimate the power of a gravitational wave that is generated.

There is no new Physics here, simply a different approach or formulation of the classical quadrupole equations, utilized to estimate GW power, in order to render astrophysical applications more apparent. The standard quadrupole equation, which was originally formulated by A. Einstein (1918) to compute the GW power, will be employed. The basic quadrupole approximation will be defined in terms of a change in force, $\Delta f$, over a short time interval, $\Delta t$, which is conventionally defined as a "*jerk*," a shake or an impulse. The derivation of this basic jerk equation will be accomplished by two separate analysis paths: one starting with the third derivative of the moment of inertia formulation of the quadrupole equation and the other starting with the spinning rod (or binary-star orbit) formulation of the quadrupole equation. The resulting equation will be numerically checked against the known result for the neutron-star pair pulsar PSR 1913+16. There need not be two orbiting masses to generate GWs. A. Einstein and N. Rosen (1937) show GWs generated from a single harmonic oscillator and J. Weber (1964) from a piezoelectric crystal and a single-aspherical-star jet GW source is discussed.

## 2. Derivation from the Einstein Quadrupole Formulation

As is well known (Einstein 1918) and noted specifically in a letter from G. Burdge (2000), Deputy Director for Technology and Systems of the *National Security Agency*: "Because of symmetry, the quadrupole moment can be related to a prin-

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cipal moment of inertia, $I$, of a three-dimensional tensor of the system and ... can be approximated by

$$-\frac{dE}{dt} \approx -\frac{G}{5c^5}\left(\frac{d^3I}{dt^3}\right)^2 = -5.5\times10^{-54}\left(\frac{d^3I}{dt^3}\right)^2 ."$$

or from Eq. (110.16) of L. D. Landau and E. M. Lifshitz (1975):

$$P = -\frac{dE}{dt} = \kappa\left(\frac{G}{45c^5}\right)\left(\frac{d^3 D_{\Upsilon v}}{dt^3}\right)^2 W \qquad (1)$$

or

$$P = 1.76\times10^{-52}\left(\frac{d^3I}{dt^3}\right)^2 W. \qquad (2)$$

This is Einstein's *quadrupole equation* phrased in a different fashion. In Eq. (1) $G$, the universal gravitational constant $= 6.67423\times10^{-11}\ m^3/kg\text{-}s^2$, $c$ is the speed of light $= 2.998x10^8\ ms^{-1}$ and the units in Eq. (2) are in the MKS system. In order to introduce the jerk concept concretely, let us consider the hypothetical example of a dumbbell that need not be uniformly rotating or, in fact, not rotating at all; but subjected to an impulsive force. In this case, for the dumbbell-shaped collection of masses,

$$I = \delta m\, r^2 \qquad kg\text{-}m^2 \qquad (3)$$

where $\delta m$ = mass at either end of the dumbbell, $kg$, and $r$ = the distance from a pivot out to $\delta m$, $m$, (or more exactly, the *radius of gyration* of the dumbbell). Thus

$$\frac{d^3I}{dt^3} = \delta m\left(\frac{d^3(r^2)}{dt^3}\right) = 2r\delta m\left(\frac{d^3r}{dt^3}\right) + \cdots \qquad (4)$$

Approximately, by delta differentiation,

$$2r\left\{\delta m\left(\frac{d^3r}{dt^3}\right)\right\} \cong 2r\left\{\delta m\frac{\Delta\left(\frac{d^2r}{dt^2}\right)}{\Delta t}\right\} \qquad (5)$$

and, by noting that by Newton's second law of motion:

$$f_t = \delta m\left(\frac{d^2r}{dt^2}\right), \qquad (6)$$

we have, again by delta differentiation,

$$\delta m\Delta\left(\frac{d^2r}{dt^2}\right) = \Delta f_t \qquad (7)$$

where $f_t$ = tangential (to the circular path of a rotating dumbbell) force on $\delta m$ and $\Delta f_t$ is the rapid increase in $f_t$ over time $\Delta t$ (the jerk or shake or impulse). The third derivative of $I$ is, therefore, approximated by

$$\frac{d^3I}{dt^3} \cong 2r\frac{\Delta f_t}{\Delta t}, \qquad (8)$$

in which $\Delta f_t$ is the nearly instantaneous increase in the force. The GW radiates in a figure "8" - shaped pattern (Landau & Lifshitz 1975; Baker, Davis & Woods 2005) from the midpoint of the dumbbell in both directions along the axis of the dumbbell pivot. In summary, by substituting Eq. (8) into Eq. (2),

$$P = 1.76\times10^{-52}\left(2r\frac{\Delta f_t}{\Delta t}\right)^2 W, \qquad (9)$$

which is the jerk formulation of the quadrupole equation.

## 3. Derivation from the Spinning-Rod Quadrupole Formulation

An alternative derivation of Eq. (9) is as follows: From Eq. (1) of J. Weber (1964) one has for Einstein's 1918 formulation of the gravitational-wave (GW) radiated power of a rod spinning about an axis through its midpoint having a moment of inertia, $I$, $kg\text{-}m^2$, and an angular rate, $\omega$, radians/$s$, [also please see, for example, Misner, Thorne, Wheeler (1973) in which $I$ in the kernel of the quadrupole equation also takes on its classical-physics meaning of an ordinary moment of inertia]:

$$P = 32G\frac{I^2\omega^6}{5c^5} = G\frac{(I\omega^3)^2}{5\left(\frac{c}{2}\right)^5}W, \qquad (10)$$

or, with $I = r^2m$ ($r$ being the radius of gyration of the rod),

$$\begin{aligned} P &= 1.76\times10^{-52}\left(I\omega^3\right)^2 \\ &= 1.76\times10^{-52}(r\{rm\omega^2\}\omega)^2W \end{aligned} \qquad (11)$$

where $\{rm\omega^2\}$ can be associated with the magnitude of the rod's centrifugal-force vector, $\boldsymbol{f_{cf}}$. Equation (11) is the *same equation as that given for two bodies on a circular orbit* by L. D. Landau and E. M. Lifshitz (1975) ($I = \mu r^2$ in their notation) where $\omega = n$, the orbital mean motion, radians/s. The $\boldsymbol{f_{cf}}$ vector reverses every half period at *twice the angular rate of the rod* [and a $\boldsymbol{f_{cf}}$ component's magnitude completes one complete period in half the rod's period as in J. Weber (1964)]. Thus the GW frequency is $2(\omega/2\pi)$, where $\omega$ is in radians/$s$. The change in the centrifugal-force vector itself (essentially a "jerk" when divided by a time interval) is a differential vector at right angles to the $\boldsymbol{f_{cf}}$ vector and directed tangentially along the arc that the rod ends move through. The differential change in, for example, the $x$-component of the change in centrifugal force, $\Delta f_{cfx}$, is $f_{cfx}\Delta\theta$ and the change in the $y$-component, $\Delta f_{cfy}$, is $f_{cfy}\Delta\theta$, where $\theta$ is the central angle of the rotating rod in radians. By delta differentiation of $f_{cf}^2 = f_{cfx}^2 + f_{cfy}^2$,

$$f_{cf}\Delta f_{cf} = f_{cfx}\Delta f_{cfx} + f_{cfy}\Delta f_{cfy} \qquad (12)$$

and when one associates the components $\Delta f_{cfx,y}$ with $f_{cfx,y}\Delta\theta$ and, after dividing by $\Delta t'$ ($t'$ being spinning-rod time), and noting that $\Delta\theta/\Delta t' = \omega$,

$$\frac{f_{cf}\Delta f_{cf}}{\Delta t'} = (f_{cfx}^2 + f_{cfy}^2)\omega. \qquad (13)$$

Thus $\Delta f_{cf}/\Delta t' = f_{cf}\omega$; but $\Delta t' = \frac{1}{2}\Delta t$ since the period of the GW is half the period of the rod, so that:

$$2\frac{\Delta f_{cf}}{\Delta t} = f_{cf}\omega, \qquad (14)$$

but $f_{cf} = \{rm\omega^2\}$ so

$$2\frac{\Delta f_{cf}}{\Delta t} = \{rm\omega^2\}\omega, \qquad (15)$$

and substituting Eq. (15) into Eq. (11) yields

$$P = 1.76\times10^{-52}\left(2r\frac{\Delta f_{cf}}{\Delta t}\right)^2 W, \qquad (16)$$

where $(2r\Delta f_{cf}/\Delta t)^2)$ is the kernel of the quadrupole approximation equation and $\Delta f_{cf}/\Delta t$ is, again, the *jerk*. Equation

(16) is *identical* to Eq. (9), but arrived at by a different analysis path. Equation (9), like Eqs. (2), (10), (11) and (16), are approximations for GW power and may only hold accurately for $r \ll \lambda_{GW}$ and for speeds of the GW generator components far less than the speed of light, $c$. Please see, for example, A. Pais (1982) and K. S. Thorne (1989). On the other hand, L.P. Grishchuk (2003) suggested that the requirement that $r \ll \lambda_{GW}$ may ***not*** be a stringent or even a necessary one for the quadrupole approximation to GW power to hold. As K. S. Thorne (1989) states "…the quadrupole formalism typically is accurate to within factors of order 2 even for sources with sizes of (the) order (of) a reduced (GW) wavelength …" Whether the quadrupole approximation to the power of gravitational wave generation holds or not does not necessarily imply that no GWs are generated by an impulsive force acting on a pair of masses or that the power does not increase with the distance, $2r$, between the radiating masses. The quadrupole formalism may still provide order-of-magnitude estimates perhaps augmented by higher-order octupole, hexadecapole, etc. modes of pulsation or jerk and possibly reduced at the GW focus by diffraction. It is a problem deserving study in the future.

## 4. Validation Based Upon the Orbit of PSR 1913+16

As a numerical validation of Eqs. (9) and (16), that is a validation of the use of a jerk to estimate gravitational-wave power, let us utilize the approach for computing the gravitational-radiation power of the neutron-star pair PSR 1913+16 observed by J. H. Taylor (1994) and Hulse to demonstrate the existence of GWs. The theoretical GW power computed by means of the classical formulation of the quadrupole equation agreed with observation so if the jerk formulation of the GW power agrees with the classical manner of calculating the power, then the jerk formulation is validated. In the case of a binary star pair such as PSR 1913+16, the magnitude of the GW power, $P$, is computed from the classical quadrupole equation, which for two masses on orbit about one another is given, for example, by an equation on p. 356 of L. D. Landau and E. M. Lifshitz (1975) or P. C. Peters and J. Mathews (1963), the *time-constant factor* in the equation for $P$ is

$$\frac{8G^4 m_1^2 m_2^2 \mu}{15c^5} \tag{17}$$

where $m_1$ and $m_2$ are the masses of the two stars. The *time-variable factor* in $P$ is a function of the true anomaly, $v$, and orbital eccentricity, $e$, as given in L. D. Landau and E. M. Lifshitz (1975) is:

$$(1 + e\cos y)^4 \frac{\left[1 + \frac{e\cos v}{12}\right]^2 + e^2 \sin^2 v}{(a[1-e^2])^5}. \tag{18}$$

In conventional Astrodynamic or celestial-mechanics notation (Herrick 1971) this factor [i.e., Eq. (18)] is

$$\frac{p}{r^6} + \frac{\left(\frac{dr}{d\tau}\right)^2}{12\mu r^4} \tag{19}$$

where $p$ is the orbital "parameter" or semilatus rectum ($= a[1-e^2]$) in astronomical units or A.U.s, $r$ is the radial distance between the two masses, $\tau$ is the characteristic time measured in conventional celestial mechanics units of $k_s$days or in units of $5.022 \times 10^6$ $s$ for a heliocentric-unit system (utilized by Taylor and others for PSR 1913+16), $\mu$ is the sum of the two masses, i.e., $\mu = m_1 + m_2$ solar masses, and as usual $G$ is the universal gravitational constant and $c$ is the speed of light. Note that one A.U. = $1.496 \times 10^{11}$ $m$. The GW power radiated, $P$, which causes a perturbation in the semi-major axis, $a$, (and an attendant observed secular decrease in the orbital period) is obtained by integrating the time-variable factor, Eq. (19), over the orbital period using the mean anomaly, $M$, as independent variable, which is directly proportional to the time (that is, $M = n\{t - T\}$, where $n$ is the mean motion ($\omega$ in L. D. Landau and E. M. Lifshitz's (1975) notation), $n = 2\pi$/Period$= 2\pi/(2.79 \times 10^4) = 2.25 \times 10^{-4}$ $s^{-1}$, and $T$ is the time of periastron passage).

The $x$ and $y$ average delta centrifugal force (or centrifugal-force jerks) component(s), $\Delta f_{cfx,y}$ are both

$$\begin{aligned} man^2 &= (5.56 \times 10^{30})(2.05 \times 10^9)(2.25 \times 10^{-4})^2 \\ &= 5.77 \times 10^{32}\ N\,. \end{aligned} \tag{20}$$

and divided by $m$ yields the average centrifugal acceleration = 103.78 $m/s^2$ = 10.6 $g$'s. At periastron, $r = q = a(1-e) = (2.05 \times 10^9)(1 - 0.641) = 7.36 \times 10^8$ $m$ (with $e = 0.641$), the centrifugal acceleration is $q(dv/d\tau)^2$ where $dv/d\tau = \sqrt{\mu p}/r^2$ (Baker 1967). In this latter case $\mu = 2.8$ solar masses, $a =$ 2.95 solar radii = (2.95)($6.965 \times 10^8$ $m$/solar radii)/$1.496 \times 10^{11}$ $m$/AU = 0.01373 AU, $p = a[1-e^2] = 0.01373(1 - 0.4109) = 0.00809$ AU, and $q = r = 7.36 \times 10^8$ $m$/$1.496 \times 10^{11}$ $m$/AU = 0.00495 AU. After inserting these numbers we have $dv/d\tau = (\sqrt{2.8 \times 0.00809}/[0.00495]^2)/5.022 \times 10^6$ $s$/k$_s$day = $1.223 \times 10^{-3}$ radians/$s$. Thus the centrifugal acceleration at periastron of the star pair is $q(dv/dt)^2 = (7.36 \times 10^8$ $m)(1.223 \times 10^{-3}$ radians/$s)^2 = 1.101 \times 10^3$ $ms^{-2}$ = 112 $g$'s apparently *still well within the weak-field approximation of Einstein's GW equations*.

From Eq. (20) each of the components of force change, $\Delta f_{cfx,y} = 5.77 \times 10^{32}$ $N$ (multiplied by two since the centrifugal force reverses its direction each half period) and $\Delta t = \frac{1}{2}$(7.75hr × 60min × 60sec) $= 1.395 \times 10^4$ $s$ for the half period. Thus using the jerk approach:

$$\begin{aligned} P &= 1.76 \times 10^{-52} \left\{ \left(2r\frac{\Delta f_{cfx}}{\Delta t}\right)^2 + \left(2r\frac{\Delta f_{cfy}}{\Delta t}\right)^2 \right\} \\ &= 1.76 \times 10^{-52} \left(\frac{2 \times 2.05 \times 10^9 \times 5.77 \times 10^{32}}{1.395 \times 10^4}\right)^2 \times 2 \\ &= 10.1 \times 10^{24}\ W \end{aligned} \tag{21}$$

versus the result of $9.296 \times 10^{24}$ $W$ using L. D. Landau and E. M. Lifshitz's more exact two-body-orbit formulation given by Eqs. (1.1) and (1.2) of R. M. L. Baker, Jr. (1967) integrated using the mean anomaly not the true anomaly as independent variable. The most stunning closeness of the agreement is, of course, fortuitous since due to orbital eccentricity there is not complete symmetry among the $\Delta f_{cfx,y}$ components around the orbit and there are small errors inherent in the approximations of Eqs. (4), (5) and (8) leading to Eq. (16). Nevertheless, since the results for GW power are so close, orbital-mechanic formulation compared to the utilization of a jerk, *the correctness of the jerk formulation is demonstrated.*

## 5. Discussion

Before considering potential applications, it should be recognized that gravitational forces are *not* required for the generation of GWs. As J. Weber (1964) points out: "The non-gravitational forces play a decisive role in methods for detection and generation of gravitational waves ..." As an example, M. E. Gertsenshtein (1962), in his pioneering paper, established that electromagnetic waves could generate GWs directly and *vice versa*.

There are at least two potential astrophysical sources of GWs whose power can be easily estimated by Eqs. (9) or (16). The first is a single jet similar to that produced by Cygnus X-1 (Fig. 2 of Gallo et al. 2005), that could produce an impulsive force, $\Delta f/\Delta t$, acting on one member of the X-ray binary, that are $2r$ apart ($r$ being the orbital radial distance or the radius of gyration of the pair). The second potential GW source would be caused by other single-direction (not from each pole) stellar-jet-produced impulsive forces, which could nearly simultaneously act on two stellar sources that are widely separated but still "connected" by gravity, perhaps located on opposite sides of a galaxy or in different galaxies like a giant pinwheel. [The midpoint or GW focus at the two star's center of mass can be unrelated to any stellar-system's (that is, a large number of stellar masses such as a galaxy's) center of mass.] The impulsive forces need not be exactly opposite and coplanar. They could be 60 degrees or so apart and still have significant force components that are opposite and coplanar. Also they need not be exactly simultaneous events. For example, if they are 100,000 light years or $10^{21}$ $m$ apart and the GW focus, midway between the stars (Landau & Lifshitz 1975; Baker et al. 2005), is shifted 10% of the way towards the "late" jet-emitting star, then the time differential could be

$$\Delta t = \frac{0.1 \times 10^{21}}{c} = 3.3 \times 10^{12}\ s \text{ or } 106,000\ y. \qquad (22)$$

These examples are very arbitrary and hypothetical, but serve to illustrate that it is valuable to keep the jerk-formulation of the quadrupole equation in the repertoire of analytical tools for the estimation of the power of GWs. The simplicity of the new quadrupole formulation arises due to the fact that only estimates of three quantities: (1) the radius of gyration of a massive stellar object, exhibiting a dumbbell-like, rod-like or aspherical form, or half of the distance between two GW stellar radiators, $r$, (2) the magnitude of a transient force $\Delta f$, (or its component perpendicular to the axis of rotation) and (3) the length of time that the transient force acts, $\Delta t$, are required to calculate an approximation to the generated GW power. [More exactly, $\Delta f$ is the component of the impulsive force, $\Delta f_t$, tangent to the circle of revolution of the star pair (i.e., orbit), rod or dumbbell-shaped system of masses such as dumbbell-shaped or aspherical individual star with a single jet impulse. In the case of a prolate aspherically shaped star, the radius of gyration in Eq. (9) is that of an equivalent dumbbell as indicated W. B. Klemperer and R. M. L. Baker, Jr. (1957) e.g., for a perfectly spherical star it would be zero.] No elaborate analytical calculation or sophisticated computer program, i.e., numerical relativity (Ashby et al. 2000), is required. It is also noted that the GW frequencies, which would be dependent on jet-emission time, $\Delta t$, (pulse or wavelength is $c\Delta t$ and effective frequency is $1/\Delta t$) would be quite low and probably only detectable by the proposed space-based LISA array. At the high-frequency end of the GW spectrum the quadrupole formalism of Eq. (9) could be applied to the laboratory generation of high-frequency gravitational waves or HFGWs (Baker, Woods & Li 2006, and Baker 2004).

## References


Ashby, S. F., Foster, I., Lattimer, N., Parashar, M., Saylor, P., Schutz, A., Seidel, E., Suen, W., Swesty, F. D., Clifford, M.: 2000, Final Report, NASA CAN NCCS5-153, October 15, NASA Headquarters, Washington DC, USA

Baker, R. M. L., Jr.: 1967, Astrodynamics: Applications and Advanced Topics, Academic Press, New York, 13

Baker, R. M. L., Jr.: 2004, U.S .Pat. No. 6417597B1 and P.R. China 10055882.2

Baker, R. M. L., Jr., Davis, E. W., Woods, R. C.: 2005, in: M.S. El-Genk (ed.), Space Technology and Applications International Forum (STAIF-2005), AIP Conf. Proc. 746, 1315 (published by the American Institute of Physics, Melville, NY)

Baker, R. M. L., Jr., Woods, R. C. Li, F.: 2006, in: M.S. El-Genk (ed.), Space Technology and Applications International Forum (STAIF-2006), AIP Conf. Proc. 813, 1280 (published by the American Institute of Physics, Melville, NY)

Burdge, G.: 2000, written communication from the United States National Security Agency to R. M. L. Baker, Jr., dated January 19

Einstein, A.: 1918, Sitzungsberichte, Preussische Akademie der Wisserschaften, 154

Einstein, A., Rosen, N.: 1937, J. of Franklin Inst. 223, 43-54

Gallo, E., Fender, R., Kaiser, C., Russell, D., Morganti, R., Oosterloo, T., Heinz, S..: 2005, Natur 436, 819

Gertsenshtein, M. E.:1962, SJETP 14, 85

Grishchuk, L. P.: 2003, personal comments made at The MITRE Corporation Gravitational Wave Conference, McLean, Virginia, USA, May 8

Herrick, S.: 1971, Astrodynamics Volume 1, Van Nostrand Reinhold, 60 61

Klemperer, W. B., Baker, R. M. L., Jr.: 1957, AsAc., Fasc. 1, 16-27

Landau, L. D., Lifshitz, E. M.: 1975, The Classical Theory of Fields, Fourth Revised English Edition, Pergamon Press, 348, 349, 355-357

Misner, C. W., Thorne, K., Wheeler, J. A.: 1973, Gravitation, W. H. Freeman and Company, NY, 979 , 980

Pais, A.: 1982, Subtle is the Lord, The Science and the Life of Albert Einstein, Oxford University Press, 242, 280, 384

Peters, P. C., Mathews, J.: 1963, PhRv.131, 435-440

Taylor, J.H.: 1994, RvMP. 66, 711-719

Thorne, K. S.: 1989, Chapter 9 of 300 Years of Gravitation,. Hawking, S., Israel, W. eds., Cambridge University Press, Cambridge, 347, 357

Weber, J.: 1964, in Gravitation and Relativity, Benjamin, New York, NY, Chapter 5, 90-105